Co-evolutionary dynamics in social networks: A case study of Twitter

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***Abstract*—Complex networks often exhibit co-evolutionary dynamics, meaning that the network topology and the state of nodes or links are coupled, affecting each other in overlapping time scales. We focus on the co-evolutionary dynamics of online social networks, and on Twitter in particular. Monitoring the activity of thousands of Twitter users in real-time, and tracking their followers and tweets/retweets, we propose a method to infer new retweet-driven follower relations. The formation of such relations is much more likely than the exogenous creation of new followers in the absence of any retweets. We identify the most significant factors (reciprocity and the number of retweets that a potential new follower receives) and propose a simple probabilistic model of this effect. We also discuss the implications of such co-evolutionary dynamics on the topology and function of a social network.**

I. Introduction

Online Social Networks (OSNs), such as Twitter and Facebook, have changed how individuals interact with society, how information flows between actors, and how people influence each other. These are all complex dynamic processes that are now widely studied empirically and in large scale, thanks to the availability of data from OSNs. Most OSN studies focus on one of the following two aspects of network dynamics. Dynamics *on* networks refer to changes in the state of network nodes or links considering a static topology [6, 39]. Dynamics *of* networks, on the other hand, refer to changes in the topology of a network, without explicitly modeling its underlying causes [24]. As noted by Gross and Blasius in [13], however, real OSNs typically exhibit both types of dynamics, forming an adaptive, or co-evolutionary, system in which the network topology and the state of nodes/links affect each other through a (rather poorly understood) feedback loop.

Dynamic processes in OSNs, such as information diffusion or influence, are obviously affected by the underlying network topology, but they also have the power to affect that topology. For instance, users may decide to add or drop a "friendship" or "follower" relation depending on what the potential "friend" or "followee" has recently said or done in the context of that OSN. Previous empirical or modeling OSN studies often choose to ignore such co-evolutionary dynamics, mostly for simplicity, assuming a static network topology, or assuming that the topology and node/link states are decoupled and evolve in separate time scales [25].

In this paper, we focus on co-evolutionary dynamics in the context of Twitter. Twitter users create *follower-followee* relations with each other. A directed link from a user R to a user S, denoted by $R \rightarrow S$, means that R is a follower of S, receiving S's tweets; S is referred to as a followee of R. R can choose to propagate a tweet of S to her own followers, denoted by $F(R)$, creating a *retweet*. When a follower $L \in F(R)$ receives a retweet of S through R, L can choose to add S to her followers. We call this sequence a *Tweet-Retweet-Follow (TRF)* event, and refer to its three main actors as *Speaker S*, *Repeater R*, and *Listener L*. TRF events represent a clear case of co-evolutionary dynamics: information propagation (tweet-retweet) causes a topology change (new follower).

Figure 1 shows this sequence of events for the simplest TRF case in which $R \rightarrow S$ and $L \rightarrow R$. In general, the Repeater R may not be a follower of S but she may receive S's tweet through a cascade of retweets. Additionally, the Listener L may receive multiple retweets of S from the same or from different Repeaters. The contributions of this study are:

1) We propose a measurement approach to detect TRF events, based on near real-time monitoring of a Speaker's activity and followers.
2) We show that the formation of new follower relations through TRF events is orders of magnitude more likely than the exogenous arrival of new followers in the absence of any retweets.
3) We identify the most significant factors for the likelihood of a TRF event: reciprocity (i.e., is Speaker S already following Listener L?), number of received retweets (i.e., how many retweets of S were received at L during a given time interval Δ), and of course the interval Δ itself.
4) We propose a simple but accurate two-parameter model to capture the probability of TRF events.
5) We discuss the implications of TRF events in the structure and function of social networks.

II. Related Work

Preferential attachment [7] is a common way to think about the formation of new ties in a social network. It is based on the idea that it is more likely for well-connected people to attract

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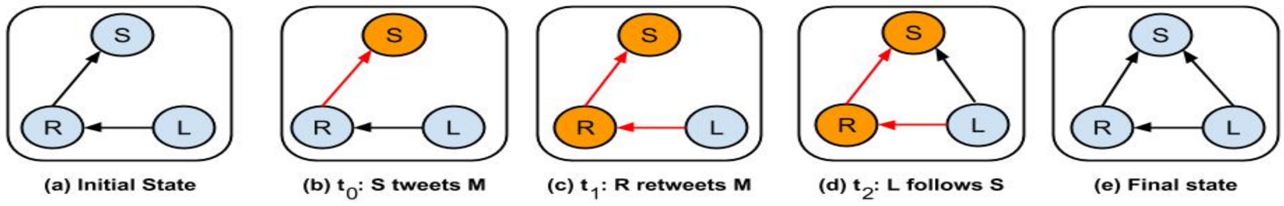


Fig. 1. Network co-evolution: a Tweet-Retweet-Follow event.

new ties. Subsequent research provided a deeper understanding by exploring mechanisms such as user similarity [17, 28] and directed closure [11, 22, 31]. For instance, Romero and Kleinberg [31] studied the *directed closure process* in Twitter. This process states that there is an increased likelihood for a node A to follow a node C if there already exists a direct path of length two from C to A. They showed that this process is taking place at a significantly higher rate than what would be expected by chance, but this rate also varies significantly among different users. Here, we identify TRF events as a plausible mechanism for the emergence of directed closure.

Golder and Yardi conducted a user study to identify structural predictors for tie formation in Twitter. Their results show that lack of transitivity has a negative effect in link prediction [11]. Hopcroft et al. examined the question: "when you follow a particular user, how likely will she follow you back?" [17]. They showed that geographic distance and homophily are good predictors of follow-back ("reciprocal") relations. Our work confirms that reciprocity amplifies significantly the likelihood of TRF events. Leskovec et al. studied the network evolution of four social networks and observed that most edges are local, "closing triangles" in particular [22].

The literature on co-evolutionary dynamics has relied mostly on abstract models so far, without sufficient empirical validation. For instance, Kosma and Barrat examined how the topology of an adaptive network of interacting agents and of the agents' opinions can influence each other [19]. When agents rewire their links in a way that depends on the opinions of their neighbors, the result can be either a large number of small clusters, making global consensus difficult, or a highly connected but polarized network. Shaw and Schwartz [33] examined the effects of vaccination in static versus adaptive networks. Interestingly, they show that vaccination is much more effective in adaptive networks.Volz and Mayers studied epidemics in dynamic contact networks and showed that the rate at which contacts are initiated and terminated affects the disease reproductive ratio [40]. Rocha et al. simulated epidemics in an empirical spatio-temporal network of sexual contacts [30], showing that dynamic network effects accelerate epidemic outbreaks. Perra et al. studied the effect of time-varying networks in random walks and search processes [29]. The behavior of both processes was found to be "strikingly different" compared to their behavior in static networks.

The most relevant prior work, by Weng et al., analyzed the complete graph and activity of *Yahoo! Meme*, to identify the effect of information diffusion on the evolution of the underlying network [42]. They show that information diffusion causes about 24% of the new links, and that the likelihood of a new link from a user X to a user Y increases with the number of Y's posts seen by X. More recently, Myers and Leskovec showed that Twitter users gain or loose bursts of followers soon after their tweet activity event [27]. These bursts increase both the density of connections between a user's followers and the similarity of a user with her followers. Similarly to our work, they show that 21% of all new follows are formed by users who recently saw a retweet of the target user.

III. Data Collection

To identify TRF events we need to observe the appearance of a new follower link from an arbitrary Listener L to a monitored Speaker S, shortly after L has received a retweet of S through a Repeater R. This requires information about both the time of the retweet(s) as well as the time the new follower link has appeared. The Twitter API, though extended in functionality, does not provide information about the creation time of follower relations. Furthermore, existing link creation time inference methods [26] are not applicable in our study because they cannot be used in real-time. To retrieve (near) real-time timing we have implemented a Twitter data retrieval system that periodically checks for new followers and retweets in a given set of Speakers. We explain each step of the process in the following paragraphs.

Selection of active Speakers: We obtain a number of active Twitter users as potential Speakers through a stratified sampling method. It has been reported that about 25% of Twitter users have never posted any messages [1] and that most users check their Twitter feeds rarely [21]. A random user selection process would most likely visit a number of users without recent posts, wasting a large number of our limited Twitter API calls. The adopted sampling method ensures that we monitor users that have recently posted a tweet. Specifically, we crawl the Twitter search page [2] based on a single-character search selected at random from the set of $[1-9A-Za-z]$. The search returns the latest 20 tweets containing the search term. We identify the users that posted these tweets and add them to our monitored Speakers set. For each selected Speaker, we also collect information about their "join time", number of followees, followers and posted tweets. For each observed tweet, we collect the time it was posted and the posted message.

Given this set of monitored Speakers, we look for any retweets of their tweets posted during the last two hours.We only consider retweets that are flagged as such by the Twitter API. For each retweet, we retrieve the set of followers, set of followees of the Speaker and the Repeater R at the time instant we first observed that retweet. Additionally, we collect the set of followers and followees of the Repeater at that time.

Monitoring of Speakers: The previous process results in a number of possible TRF events, whenever a follower of a Repeater receives a retweet of a monitored Speaker. To identify new followers we need to examine any changes in the Speaker's followers before and after the retweet. To do so, we retrieve the set of followers of the Speaker periodically, approximately every 5 minutes. We identify a TRF event when the set of followers of S gains a new member (Listener L) that was previously seen in the set of followers of R. At that point we log the time L was seen to follow S and calculate *TRF latency* as the time difference between the time R retweeted S and the time L followed S. If L received multiple retweets of S (as the same tweet from multiple Repeaters, multiple tweets from the same Repeater, or multiple tweets from multiple Repeaters), we assign the TRF event to the most recent retweet of S received by L. The intuition here is that the most recent tweets will appear at the top of L's inbox and they are more likely to be read than older retweets. At this point we also collect the set of followers and followees of the Listener.

Every 5 minutes, we also update the set of monitored Speakers as follows. If a selected Speaker has not posted any tweets during the last 24 hours, we stop monitoring that user and select a new Speaker using our sampling method. The reason is that most new follower relations tend to occur within few hours from the time a Speaker has been active [4, 41].

a) Data collection system: Due to the complexity and the real-time nature of our data collection process, we need a large Twitter API request throughput. We used Twitter's API 1.0, which limits users to 350 API requests per hour. To increase this request throughput we use a large number of distributed hosts, provided by PlanetLab, as proxies for accessing Twitter [9]. Our collection process is coordinated by a "dispatcher" application located at Georgia Tech. The dispatcher decides what data are required at any point in time and instructs a number of "workers" to request that data from Twitter. Each worker is assigned a single Planetlab host that routes API requests to Twitter. When a worker runs out of requests it deactivates itself and notifies the dispatcher. At that point the dispatcher generates a new worker, providing it with a fresh request workload.

We divide the data collection process to small independent processes, each of them requiring the smallest possible number of requests. In this way, we partition different parts of the Speaker monitoring process to a number of workers, speeding up the collection process. For instance, when requesting an update for a Speaker, the retrieval of tweets, retweets and follower sets are executed through different Planetlab hosts. Further, we limit the number of concurrently monitored Speakers to 500 to avoid overloading both Twitter and our collection system.

b) Bot-filtering: A major concern for any Twitter dataset is to avoid bots. Such accounts act differently than most regular Twitter users, biasing the analysis. To identify and remove bot accounts from our dataset we revisited each account three months after the initial data collection to check which of those accounts have been suspended by Twitter. This practice has been used by Thomas et al. [38] as "ground truth" for the Twitter bot detection problem. Further, it has been reported that only few bots survive Twitter's policies for more than a week [34]. In our data, about 1% of the observed users were suspended by Twitter (uniformly distributed across Speakers, Repeaters, and Listeners), accounting for roughly 10% of the observed TRF events.

Dataset-1 To estimate the exogenous and endogenous probabilities (Section IV) we use a small-scale dataset (compared to the dataset used in the rest of the paper). Specifically, we monitor 200 unique Twitter users (Speakers) for a period of 10 days. For each Speaker, we collect periodically (every 30 minutes) her Twitter timeline, tweets and retweets, along with the list of her followers. We also collect the followers of every follower of the 200 monitored Speakers. Based on this dataset we can observe all Tweet-Retweet (TR) events for every monitored Speaker over the course of 10 days, and so we can ask whether a Speaker has gained one or more new followers among the set of Listeners of her retweets.

Dataset-2 In the rest of the paper we use a larger dataset. This dataset was collected during one week, from September 19 to September 25, 2012. During this time period, we collected about 300 GBytes of raw Twitter data. In this dataset we monitored 4,746 Speakers that posted 386,980 tweets. These messages were retweeted 146,867 times by 83,860 distinct Repeaters. Twitter allows users that are not following a Speaker to retweet her messages. For this reason, in Dataset-2, we do not require that the Repeaters are followers of the Speaker. After removing bot accounts, we end up with 7,451 observed TRF events. This figure represents 17% of the new follower links observed in our dataset.

IV. Endogenous versus exogenous link creation

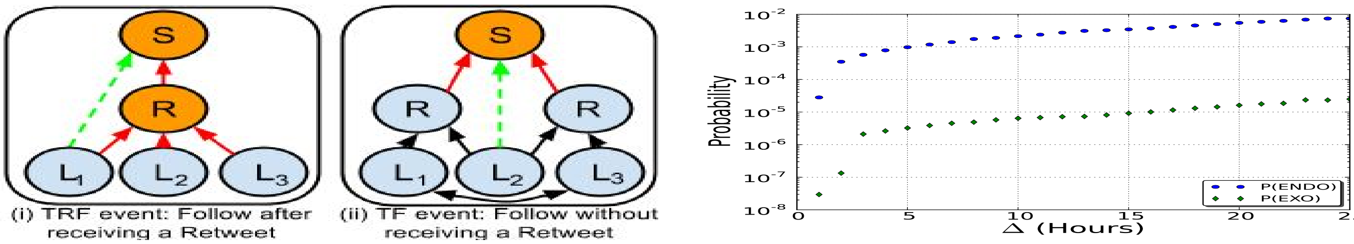
A user also gains new followers due to exogenous factors, such as Twitter's "Who to follow" service [14]. Here, we compare the likelihood with which a user gains new followers when there are no recent retweets of her messages (exogenous link creation) compared to the case that she gains new followers when at least one of her messages has been recently retweeted (endogenous link creation).

We focus here on potential new followers L of S that were already following a follower of S. That is, we only examine three-actor relations in which $L \to R$ and $R \to S$. We then ask *"is it more likely that L will follow S ($L \to S$) when L received a retweet of S through R (TRF event) or when L did not receive any retweet from her followees that follow S (TF event)?"* Figure 2(a) illustrates the TRF and TF events. Note that the difference between endogenous (TRF) and exogenous (TF) events is the retweet of S from R; the local structure and the activity of S remain the same in both cases.

We estimate the probability $P_{EXO}(\Delta)$ of exogenous new followers as follows. Consider a tweet of Speaker S at time t_s. Suppose that this tweet is not retweeted by any of the followers of S in the period $[t_s, t_s+\Delta]$. Let $\Phi(S,t_s)$ be the set of followers of followers of S that are not directly following S at t_s, i.e., $\Phi(S,t_s) = \{X : X \notin F(S,t_s), X \in F(Y,t_s), Y \in F(S,t_S)\}$. What is the fraction of these users that follow S by time $t_s + \Delta$?

$$P_{EXO}(\Delta) = \frac{|L : L \in \Phi(S,t_s), L \in F(S,t_s+\Delta)|}{|\Phi(S,t_s)|} \tag{1}$$

Similarly, we estimate the probability $P_{ENDO}(\Delta)$ of endogenous new followers as follows. Consider again a tweet of Speaker S at time t_s but suppose that this message has been retweeted by a specific follower of S, referred to as Repeater R, at time $t_r > t_s$. Let $\Phi_R(S,t_r)$ be the subset of $\Phi(S,t_r)$ that



(a) (b)

Fig. 2. (a) Controlling for the structural relation between S, R and L and for the activity of S allows us to compare the likelihood of a new follower L when L received a retweet of S (i) compared to the case that L did not receive a retweet of S (ii). The arrow direction shows who follows whom. Orange nodes represent tweet or retweet activity. Red edges show the extent of information propagation. Green dashed edges show new follower links. (b) Probability that a Speaker S gains at least one new follower L within an interval Δ from the time of a tweet (TF) or retweet (TRF) of S. The Listener L is not a follower of S at the time of the tweet (TF) or retweet (TRF).

includes only followers of R. What is the fraction of these users that follow S by time $t_r+\Delta$?

$$P_{ENDO}(\Delta) = \frac{|L : L \in \Phi_R(S,t_r), L \in F(S,t_r+\Delta)|}{|\Phi_R(S,t_r)|} \tag{2}$$

In a small-scale dataset (Dataset-1), we observed 4,945 new followers for the 200 monitored Speakers over 10 days. TRF events accounted for 42% of these new links. This shows that TRF events are rather infrequent, compared to tweets and retweets, but they are responsible for a large percentage of the new links in Twitter.

Figure 2(b) compares the two probabilities for increasing values of Δ, averaged across all TF and TRF events in our dataset. We omit confidence intervals because they are too narrow. Note that the probability of endogenous new followers is consistently much higher than the probability of exogenous new followers. Especially for short Δ (up to 2 hours), P_{ENDO} is three orders of magnitude higher than P_{EXO}. The difference drops to two orders of magnitude and remains stable even for values of Δ larger than 24 hours.

Please note that the previous comparison does not prove causality: *we cannot be certain whether a user L decided to follow S because she received a retweet of S.* However, if L had not received that retweet it would be 100-1000 times less likely that she would follow S within a given time interval.

Figure 2(b) shows that P_{ENDO} increases significantly as Δ increases to about 24 hours. After that point, P_{ENDO} saturates to a value that is about 10^{-2}. It can be argued that this underestimates the actual TRF probability. The reason is that a large fraction of Twitter users are either completely inactive or they do not visit Twitter often. Recent statistics report that only 20% of registered users visit Twitter at least once per month [35]. Additionally, a report from Pew Internet [3] in 2010 reported that only 36% of Twitter users check their inbox at least once a day.

V. TRF CHARACTERISTICS

The previous analysis verifies our initial intuition that the likelihood with which a user L follows a user S greatly increases when L receives a retweet of S. Furthermore, this likelihood is also affected by the length of the interval between the retweet and the time L observed that retweet. We now give a more precise definition of Tweet-Retweet-Follow events. We say that a Tweet-Retweet-Follow event between users S, R, and L, where R might not be a direct follower of S, occurs when we observe the following sequence of events:

(a) S tweets a message M at time t_s,
(b) A user R retweets M at some time $t_r > t_s$,
(c) A user L, who is a follower of R (i.e. $L \rightarrow R$) at t_r but not a follower of S, follows S by time t_l, where $t_l \in [t_r, t_r+\Delta]$.

We collected a larger dataset (Dataset-2) that we use to analyze and model TRF events. In this dataset we observe 7,451 TRF events, which represent 17% of the observed new follower relations.

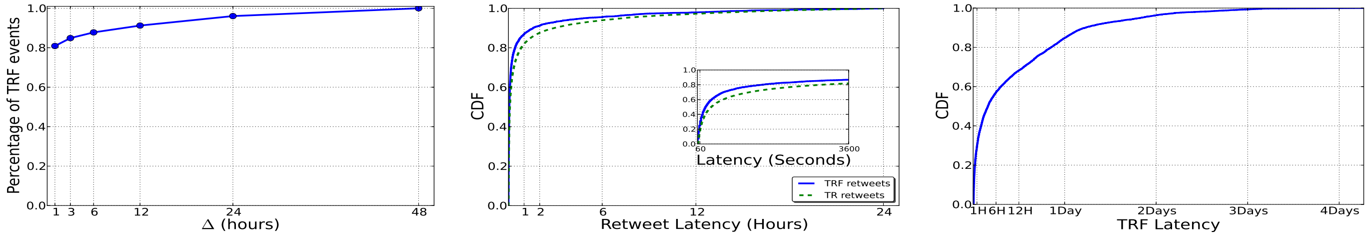
Δ is the only parameter in this definition and it affects the likelihood of TRF events. Figure 3(a) shows the percentage of identified TRF events as a function of the parameter Δ. As expected, the number of TRF events increases with Δ but most of them occur within 24 hours from the corresponding retweet.

Retweet latency: Figure 3(b) distinguishes between retweets that resulted in at least one TRF event (TRF retweets) and retweets that did not result in a TRF event (TR retweets). The analysis of these retweet events shows that more than 90% of them occur in less than an hour from the corresponding tweet; we refer to this time interval as *retweet latency*. This result supports the idea that "retweeting users" tend to act soon after new information becomes available.

TRF latency: We observe new $L \rightarrow S$ relations even 4 days after L has received a retweet of S, as shown in Figure 3(c). However, more than 80% of the TRF events occur in less than 24 hours after the retweet. Unless stated otherwise, in the rest of this paper we set Δ=24 hours.

a) TRF probability : For each monitored Speaker, we collect at each sampling instant her list of followers $F(S)$, tweets, retweets, Repeaters and the set of followers for each Repeater $F(R)$. We then identify the set of *Tweet-Retweet (TR) events* for each retweet of Speaker S: $TR(S,R,L,t_r,I_\Delta)$. A TR event denotes that Listener L received a message of S at time t_r through a retweet by Repeater R. The indicator variable I_Δ is 1 if L followed S during a time period of length Δ after t_r.

We could define the TRF probability as the fraction of TR events for which I_Δ=1. This calculation, however, does not consider that a Listener may receive multiple retweets (of the same or different tweets) of that Speaker. It would not be



(a) (b) (c)

Fig. 3. (a) Percentage of identified TRF events as function of Δ. (b) Retweet latency for all observed retweets. We plot separately retweets that lead to a TRF event, and retweets that do not (TR retweets). (c) Delay between the time of a retweet of Speaker S and the time the Listener L follows S.

realistic to assume that the Listener will decide whether to follow the Speaker immediately after each retweet. Typically, users do not read each tweet immediately when it is generated, nor they have an infinite attention span that would allow them to consider all tweets in their inbox [41]. It is more reasonable to expect that each time a user opens her inbox she reads several recent tweets at the same time. So, we assume that a Listener decides whether to follow a Speaker based on a group of received retweets that were recently received.

Specifically, we group TR events into *Retweet Groups (RG)* as follows. Each RG is represented as $RG(S,L,t_r,n,I_\Delta)$, where S and L are the Speaker and Listener, respectively, t_r is the timestamp of the first retweet in that group, and n is the number of retweets of S received by L during the time window $<t_r,t_r+\Delta>$. Note that these retweets may be generated by different Repeaters. The indicator variable I_Δ is 1 if L followed S by the end of the previous time interval. If L followed S at time $t_r \leq t \leq t_r+\Delta$, the corresponding RG includes only those retweets received by L before t; any subsequent retweets are ignored because L already follows S.

Based on this Retweet Grouping method, we calculate the TRF probability $P_{TRF}(\Delta)$ as the fraction of RGs for which I_Δ=1.

b) Factors that affect the TRF probability: We now examine a number of factors that may affect the TRF probability. The small magnitude of the TRF probability makes the identification of important factors more challenging [15]; the following results, however, are given with satisfactory statistical significance (see p-values in Table I).

Table I lists the structural and informational factors (features) we consider. We use logistic regression to analyze how these features correlate with the TRF probability. Based on (3), we estimate the correlation coefficient κ_i for each factor X_i. κ_i denotes the effect of X_i to the "odds" of TRF events,

$$ln\left(\frac{P_{TRF}}{1-P_{TRF}}\right) = \kappa_0 + \sum_{i=1}^{n}\kappa_i X_i \tag{3}$$

Table I shows the odds ratio and the corresponding 95% confidence interval for each feature. An odds ratio ρ represents a $\rho \times P_{TRF}$ increase in the TRF probability for every unit increase of the corresponding feature. Thus, odds ratios close to 1 suggest that those features have no major effect on the TRF probability. Table I shows that all odds ratios are statistically significant ($p < 0.01$).

The "Twitter age" of the Speaker, the number of followers and followees (factors that were previously shown to correlate

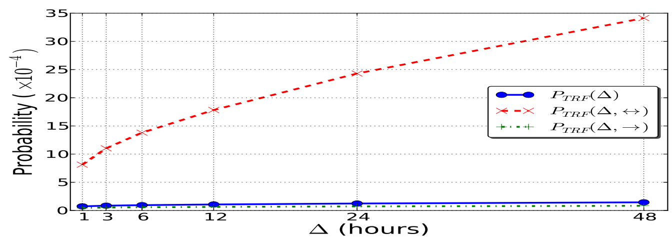


Fig. 4. $P_{TRF}(\Delta)$, Reciprocal $P_{TRF}(\Delta,\leftrightarrow)$ and Non-reciprocal $P_{TRF}(\Delta,\rightarrow)$.

with Twitter activity) as well as the tweeting [18, 21] and retweeting [36] rate of the Speaker, show no correlation with the TRF probability. Similar results are obtained when examining the age and number of followers or followees of the Listener. We have also examined a number of aggregated informational features, namely the Speaker's overall activity and her daily tweeting activity. Both features show no significant correlation with the TRF probability.

Reciprocity: A structural feature that examines the reverse relation between S and L, i.e., *whether S was already following L when L received one or more retweets of S*, has a large effect on the TRF probability. Reciprocity increases the probability that L will follow S by 27.3 times compared to the base TRF probability. Previous work has shown *reciprocity* to be a dominant characteristic of several online social networks such as Twitter [21], Flickr [8], and Yahoo 360 [20].

In 44% of the observed TRF events, S was following L prior to the formation of the reverse link. Figure 4 shows $P_{TRF}(\Delta)$ independent of reciprocity (solid line), when reciprocity is present (dashed line), and when reciprocity is not present (dotted line). When reciprocity is present, the TRF probability, denoted by $P_{TRF}(\Delta,\leftrightarrow)$, is one order of magnitude larger than the probability without reciprocity, denoted by P_{TRF} $(\Delta,\rightarrow)$. For $\Delta >$ 3 hours, $P_{TRF}(\Delta,\leftrightarrow)$ further increases and gradually becomes up to two orders of magnitude larger.

The large quantitative effect of reciprocity on the TRF probability implies that there may be different reasons for the formation of a link from L to S in that case. The existence of the reverse link, $S \rightarrow L$, could imply that these two users have some prior relation. They may know each other in other social contexts (online or offline) or they may belong to similar interest groups. In such cases, the retweet of S can make L aware of the existence and activity of S in the Twitter network.

Number of tweets and repeaters: Earlier social influence studies showed that the probability that an individual adopts

Δ	p	$p \times q$	p	$p \times q$
	Without reciprocity		With reciprocity	
1 hour	0.5×10^{-4}	0.12×10^{-4}	8.1×10^{-4}	7.2×10^{-4}
3 hours	0.5×10^{-4}	0.13×10^{-4}	11.0×10^{-4}	8.5×10^{-4}
6 hours	0.6×10^{-4}	0.14×10^{-4}	13.0×10^{-4}	9.3×10^{-4}
12 hours	0.6×10^{-4}	0.15×10^{-4}	17.6×10^{-4}	9.3×10^{-4}
24 hours	0.7×10^{-4}	0.16×10^{-4}	24.0×10^{-4}	10.2×10^{-4}
48 hours	0.8×10^{-4}	0.16×10^{-4}	33.1×10^{-4}	10.2×10^{-4}

TABLE II. ESTIMATED VALUE OF THE TWO MODEL PARAMETERS p AND $p \times q$

a new behavior increases with the number of her ties already engaging in that behavior [5, 6, 10, 16, 32]. Similarly, we examine whether the number of tweets and retweets of S received by L affects the TRF probability. It turns out that P_{TRF} increases with both the number of distinct tweets of S that L receives (odds ratio = 2.01), and with the number of distinct Repeaters that L received retweets from (odds ratio = 2.08).

For simplicity, we choose to aggregate the number of distinct Repeaters and the number of distinct tweets of S that L received into a single parameter: the total number n of retweets (potentially not distinct) of S that were received by L in a time period of length Δ. This new factor has high correlation with the TRF probability (odds ratio = 1.25, $p < 0.001$). Figure 5-left shows the TRF probability in the absence of reciprocity ($L \to S$) while Figure 5-right shows the TRF probability in the presence of reciprocity ($L \leftrightarrow S$), as a function of n.

c) TRF model: We now construct a simple model for the probability of TRF events. The objective of this exercise is to create a parsimonious probabilistic model that can be used in analytical or computational studies of co-evolutionary dynamics in social networks.

The model considers two independent mechanisms behind each TRF event: How many retweets n of Speaker S did the Listener L receive? And, did L actually observe (i.e., read) this group of retweets? The simplest approach is to assume, first, that the n received retweets are either observed as a group with probability p or they are completely missed, and second, that each observed retweet causes a TRF event independently and with the same probability q. Then, the probability of a TRF event after receiving at most n retweets is

$$P_{TRF}(n) = p \times (1 - (1-q)^n) \quad (4)$$

Thus, the probability of a TRF event after only one received retweet is $p \times q$. For a large number of received retweets, the TRF probability tends to the observation probability p.

As shown in Figure 5-left, the measured TRF probability $P_{TRF}(\to, n)$ without reciprocity seems to "saturate" after n exceeds about 10-20 retweets. The same trend is observed in the case of reciprocity (Figure 5-right), but the saturation appears earlier (after around 5-10 retweets). The model of (4) captures the dependency with n quite well. The parameters p and q depend on reciprocity as well as on the time window Δ, as shown in Table II. Reciprocity increases significantly both the observation probability p and the probability $p \times q$ that a single received retweet will cause a TRF event. As expected, increasing the observation time window Δ increases the observation probability. The effect of Δ on the probability $p \times q$ is weaker, especially when there is no reciprocity.

VI. IMPLICATIONS OF TRF EVENTS

Most prior work in online social networks focused either on the exogenous evolution of the topology (dynamics of network) or on influence and information diffusion on static networks (dynamics on network), ignoring the potential coupling between these two dynamics. In this paper, we considered co-evolutionary dynamics in the specific case of the Twitter online social network. Our study focused on the addition of new links through the so-called Tweet-Retweet-Follow events. We showed that TRF events, although infrequent compared to tweets or retweets, occur in practice and they are responsible for a significant fraction (about 20%) of the new edges in Twitter. Through (near) real-time monitoring of many Twitter users, we showed how to identify TRF events and investigated their temporal and statistical characteristics. More than 80% of TRF events occur in less than 24 hours after the corresponding retweet. The main factors that affect the probability of a TRF event are reciprocity and the total number of retweets received by the Listener.

We now discuss how TRF events may gradually transform the structure of a social network. We consider two fundamentally different network topologies, and discuss the implications of TRF events from the information diffusion perspective.

Effect on topologies with directed cycles: The left graph of Figure 6(a) shows a weakly connected network, which may be a subset of the Twitter topology. A directed cycle exists between some of its nodes, namely $A \to B \to D \to E \leftrightarrow C \to A$. Let us focus on the largest directed cycle in this network, i.e., in its largest Strongly Connected Component (SCC). The ties of the participating nodes may also include links to or from nodes out of this cycle, such as the $E \leftrightarrow F$ relation.

Suppose that A posts a tweet at some point in time and C decides to retweet it. Node E will receive that retweet and may follow A (TRF event). It is easy to see that, after a sufficiently large number of TRF events, the nodes of this directed cycle will form a fully connected directed graph, as shown in the right graph of Figure 6(a) (red edges denote connections created through TRF events), in which everyone is following all others. This transformation can only take place when a cycle already exists in the initial network; TRF events *cannot* create directed cycles. So, when an initial network includes a directed cycle, a sequence of TRF events may transform that cycle into a clique in which everyone can generate information that all others receive directly from the source.

Effect on hierarchical topologies: The left graph of Figure 6(b) shows a hierarchical weakly connected directed network. Again, this network may be a subset of the Twitter topology. This network contains no directed cycles, but a number of sink nodes (i.e. nodes with no outgoing edges; A and B in this example).

User F may receive a retweet of A and B through C, and she may then decide to follow them. After a sequence of TRF events, this network can then reach the topological equilibrium shown in the right graph of Figure 6(b), in which no new links can be added through TRF events. More generally, suppose that $F'(X) = \{X_1, \ldots, X_n\}$ is the set of followees of X. The set of Speakers that X may receive a retweet from can be defined recursively as $F'_U(X) = F'(X) \cup (F'_U(X_1) \cup \ldots F'_U(X_n))$; if user X does not have any followees then $F'_U(X)$ is the empty set.

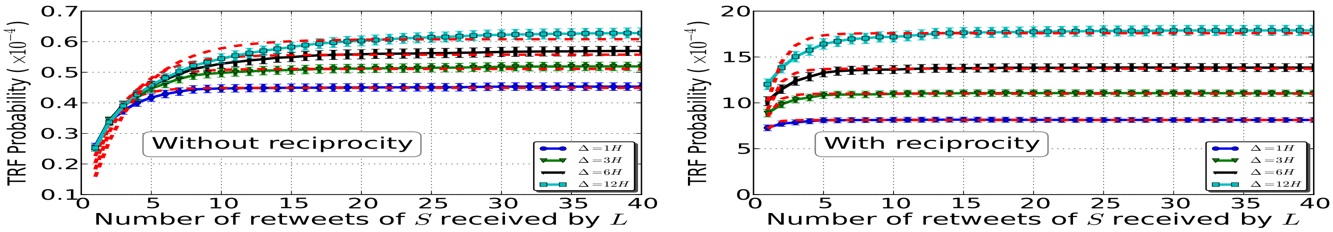
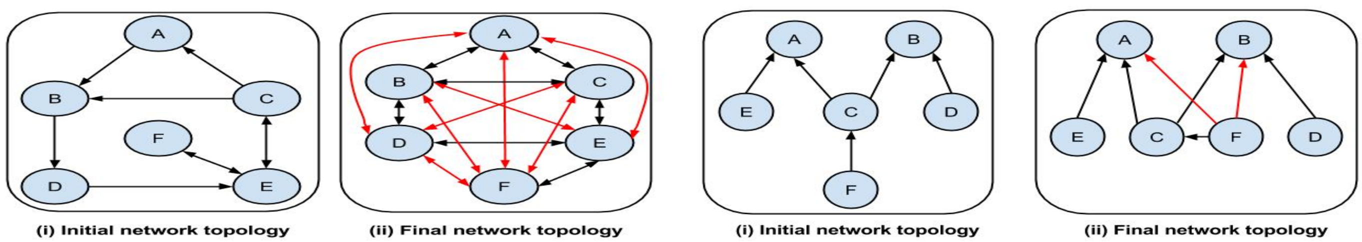


Fig. 5. Empirical (solid) and model-based (dashed) TRF probability $P_{TRF}(\rightarrow, n)$ (left) and $P_{TRF}(\leftrightarrow, n)$ (right) as a function of the number n of received retweets of S at L, for four different values of Δ

Factor	Description	Odds ratio	95% CI
	Structural Features		
$\|F(S)\|$	Number of followers of S	1.000	$[1.000, 1.000]$
$\|F'(S)\|$	Number of followees of S	0.999	$[0.999, 0.999]$
$AGE(S)$	Number of days since S joined Twitter	0.998	$[0.998, 0.998]$
$S \rightarrow L$	Reciprocity: whether the Speaker was following the Listener at the time of the TR event	27.344	$[25.663, 29.136]$
	Informational Features		
$\|ST(S)\|$	Total number of tweets of S	1.000	$[1.000, 1.000]$
$A_{rate}(S)$	Rate of S tweets per day	0.989	$[0.988, 0.991]$
$Tweets(S, L, \Delta)$	Number of distinct tweets of S received by L during period Δ	2.010	$[1.781, 2.270]$
$Retweets(S, L, \Delta)$	Number of distinct retweets of S received by L during period Δ	1.603	$[1.371, 1.873]$
$Repeaters(S, L, \Delta)$	Number of Repeaters R that L received tweets of S from during period Δ	2.076	$[1.889, 2.282]$

TABLE I. LIST OF EXAMINED FACTORS.



(a) Circular topology

(b) Hierarchical topology

Fig. 6. (a) An initial network that includes a directed cycle. A sequence of TRF events can transform this cycle to a clique, meaning that the corresponding users gradually form a tightly knit community. (b) A hierarchical initial network. A sequence of TRF events can transform this multi-layer hierarchy into a two-layer hierarchy in which each sink node is directly followed by a set of other nodes (its "sphere of influence"), while each non-sink node follows at least one sink node.

It is easy to see that, after a sufficiently large number of TRF events, a multi-layer hierarchical network will converge to a two-layer hierarchy in which every non-sink user X follows *all* users in $F'_U(X)$. Then, an initial sink node X will be followed directly by all users that had a directed path towards X in the initial network. A consequence of TRF events in such hierarchical networks is the emergence of some highly influential users that were the sink nodes in the initial network. Further, non-sink nodes will be partitioned, with the users in each partition following a distinct set of sink nodes.

The previous two topologies are obvious extremes. In practice, a given weakly connected subset of Twitter users may contain groups of nodes that form directed cycles as well as nodes that do not belong in any directed cycle. An interesting question then is: *given a weakly connected directed social network, what fraction of its nodes belong to the longest directed cycle (i.e., largest SCC) in that network?* If this fraction is large, the network resembles the example of Figure 6(a), while if it is close to zero the network is similar to the example of Figure 6(b).

We investigated the previous question based on samples of the actual Twitter topology, at least as it was measured by Kwak et al. [21] in 2010. We collected weakly connected network samples using the *Random-Walk* [23] and *Snowball* (Breadth-First-Search) [12] sampling methods. The largest SCC was determined with Tarjan's algorithm [37].

In the case of moderately large samples, between 1,000 to 1,000,000 nodes, *the largest SCC contained consistently more than 90% of the nodes.* This result suggests that the Twitter topology is closer to the network of Figure 6(a) than to the network of Figure 6(b). The creation of such large cliques, however, may require a very long time, and it may also be impractical for a user to follow thousands of other users. Consequently, we are more interested in smaller samples, including only tens or hundreds of Twitter users.

Figure 7 shows the percentage of Twitter users that are included in the SCC of small network samples, in the range of 10-1,000 nodes. Each point is the average of 1,000 samples of that size and the error bars represent 95% confidence intervals. Independent of the sampling method, the SCC typically includes the majority of the nodes even for samples of few tens of users. The SCC percentage increases to about 80-90% for networks with more than 200-400 users. These results imply that co-evolutionary dynamics, and the TRF mechanism in

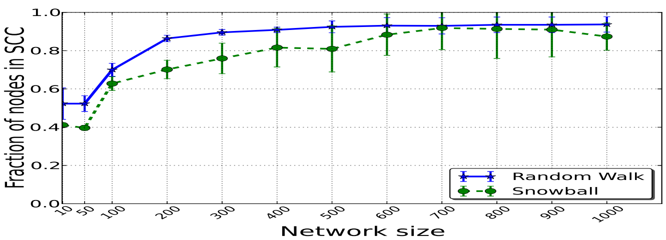


Fig. 7. Fraction of Twitter nodes in the largest SCC for different sample sizes, using two sampling methods.

particular, have the potential to gradually create very dense communities of users in which everyone is following almost everyone else, as long as the involved users are active, tweeting and retweeting information.

VII. Conclusions

Most prior work in online social networks focused either on the exogenous evolution of the topology (dynamics of network) or on influence and information diffusion on static networks (dynamics on network), ignoring the potential coupling between these two dynamics. In this paper, we considered co-evolutionary dynamics in the specific case of the Twitter online social network. Most of our study focused on the addition of new links through the so-called Tweet-Retweet-Follow events. We showed that TRF events, although infrequent compared to tweets or retweets, occur in practice and they are responsible for a significant fraction (about 20%) of the new edges in Twitter. Through (near) real-time monitoring of many Twitter users, we showed how to identify TRF events and investigated their temporal and statistical characteristics. More than 80% of TRF events occur in less than 24 hours after the corresponding retweet. The main factors that affect the probability of a TRF event are reciprocity and the total number of retweets received by the Listener. Based on these findings, we have proposed a simple probabilistic model for the probability of TRF events. We have also discussed how TRF events can affect the structure of the underlying social network. TRF events tend to transform directed cycles into cliques, creating closely knit communities of users in which everyone is following everyone else. The analysis of samples from the 2010 Twitter topology shows that weakly connected groups of more than 200-400 users contain large directed cycles that include more than 80-90% of the users. In ongoing work, we plan to use the probabilistic model proposed in this paper, extended with a model of unfollow events, to simulate co-evolutionary dynamics on a Twitter-like synthetic network.

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